Contents

Preface v

Preliminaries xi

1. **Simplicial Complexes** 1
   - 1.1 Topological Spaces 1
   - 1.2 Homotopy Equivalence and Homotopy 4
   - 1.3 Geometric Simplicial Complexes 7
   - 1.4 Triangulations 10
   - 1.5 Abstract Simplicial Complexes 13
   - 1.6 Dimension of Geometric Realizations 16
   - 1.7 Simplicial Complexes and Posets 17

2. **The Borsuk–Ulam Theorem** 21
   - 2.1 The Borsuk–Ulam Theorem in Various Guises 22
   - 2.2 A Geometric Proof 30
   - 2.3 A Discrete Version: Tucker's Lemma 35
   - 2.4 Another Proof of Tucker's Lemma 42

3. **Direct Applications of Borsuk–Ulam** 47
   - 3.1 The Ham Sandwich Theorem 47
   - 3.2 On Multicolored Partitions and Necklaces 53
   - 3.3 Kneser's Conjecture 57
   - 3.4 More General Kneser Graphs: Doľnikov's Theorem 61
   - 3.5 Gale's Lemma and Schrijver's Theorem 64

4. **A Topological Interlude** 69
   - 4.1 Quotient Spaces 69
   - 4.2 Joins (and Products) 73
   - 4.3 k-Connectedness 78
   - 4.4 Recipes for Showing k-Connectedness 80
   - 4.5 Cell Complexes 82

5. $\mathbb{Z}_2$-Maps and Nonembeddability 87
5.1 Nonembeddability Theorems: An Introduction 88
5.2 $\mathbb{Z}_2$-Spaces and $\mathbb{Z}_2$-Maps 92
5.3 The $\mathbb{Z}_2$-Index 95
5.4 Deleted Products Good 108
5.5 ... Deleted Joins Better 112
5.6 Bier Spheres and the Van Kampen–Flores Theorem 116
5.7 Sarkaria's Inequality 121
5.8 Nonembeddability and Kneser Colorings 124
5.9 A General Lower Bound for the Chromatic Number 128

6. Multiple Points of Coincidence 145
6.1 G-Spaces 145
6.2 E_nG Spaces and the G-Index 149
6.3 Deleted Joins and Deleted Products 157
6.4 The Topological Tverberg Theorem 161
6.5 Many Tverberg Partitions 165
6.6 Necklace for Many Thieves 167
6.7 $\mathbb{Z}_p$-Index, Kneser Colorings, and p-Fold Points 170
6.8 The Colored Tverberg Theorem 174

A Quick Summary 179

Hints to Selected Exercises 185

References 187

Index 203