

## Effects of solar modulation on the cosmic ray positron fraction

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### Abstract

We implemented a 2D Monte Carlo model to simulate the solar modulation of galactic cosmic rays. The model is based on the Parker's transport equation which contains diffusion, convection, particle drift and energy loss. Following the evolution in time of the solar activity, we are able to modulate a local interstellar spectrum (LIS), that we assumed isotropic beyond the termination shock, down to the Earth position inside the heliosphere. In this work we focused our attention to the cosmic ray positron fraction at energy below  $\sim 10$  GeV, showing how the particle drift processes could explain different results for AMS-01 and PAMELA. We compare our modulated spectra with observations at Earth, and then make a prediction of the cosmic ray positron fraction for the AMS-02 experiment. © 2012 COSPAR. Published by Elsevier Ltd. All rights reserved.

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### 1. Introduction

Galactic cosmic rays (GCRs) are protons, ions and leptons, produced and accelerated mainly by supernova remnants (see Blasi, 2011). GCRs remains confined in the galactic magnetic field to form a nearly isotropic flux inside the galaxy. Before reaching the Earth's orbit they enter the heliosphere, the region where the interplanetary magnetic field is carried out by the solar wind (SW). In this environment they undergo diffusion, convection, particle drift and adiabatic energy loss, resulting in a reduction of the particle's flux up to  $\sim 20$  GeV, depending on the solar activity and field polarity.

The recent accurate measurements of cosmic positrons and electrons, performed by PAMELA (Adriani et al.,

2009), show an anomalous positron excess at energies  $> 10$  GeV in comparison with the models of secondary production (see Zhang and Cheng, 2001 and Moskalenko and Strong, 1998). In the last years many papers discussing the nature of this excess have been published. Some of them suggest a dark matter signature (Yin et al., 2009); other authors invoke a primary production of electron/positron pairs by local astrophysical sources like Pulsars (Grasso et al., 2009). In this paper we do not discuss this cosmic ray positron fraction excess, since we focused on the energy interval  $\leq 10$  GeV where the same observations of CR positron fraction made by PAMELA experiment are systematically below previous measurements, like e.g. AMS-01 observations (Aguilar et al., 2007), as well as below the models of galactic secondary production.

Using our 2D Monte Carlo model (Bobik et al., 2011) we argue the reasons for this discrepancy is a solar modulation effect, that is caused by gradient and curvature drifts following changes in the magnetic field polarity. In this

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paper we first describe our modulation model, then we discuss the different behaviours of particles with opposite charge sign comparing periods with reversed polarity. Finally we compare simulation results with observations and provide also a prediction for the AMS-02 experiment.

## 2. 2D Monte Carlo model

### 2.1. Transport equations

The GCRs transport in the heliosphere is described by a Fokker–Planck equation, the so-called Parker equation (Parker, 1965):

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x_i} \left( K_{ij}^S \frac{\partial U}{\partial x_j} \right) - \frac{\partial}{\partial x_i} (v_{sw_i} U) + \frac{1}{3} \frac{\partial v_{sw_i}}{\partial x_i} \frac{\partial}{\partial T} (\rho T U) - \frac{\partial}{\partial x_i} (v_{D_i} U) \quad (1)$$

where  $U$  is the cosmic ray number density per unit interval of particle kinetic energy,  $t$  is the time,  $T$  is the kinetic energy (per nucleon),  $v_{sw_i}$  the solar wind speed along the axis  $x_i$ ,  $v_{D_i}$  is the particle drift velocity related to the antisymmetric part of diffusion tensor (Jokipii and Levy, 1977; Jokipii et al., 1977),  $K_{ij}^S$  is the symmetric part of the diffusion tensor and  $\rho = (T + 2T_0)/(T + T_0)$  (Gleeson and Axford, 1967), where  $T_0$  is particle's rest energy. This partial differential equation is equivalent to a set of ordinary stochastic differential equations (SDEs, see e.g. Gardiner, 1989) that can be integrated with Monte Carlo (MC) techniques (see e.g. Yamada et al., 1998; Gervasi et al., 1999; Zhang, 1999; Alanko-Huotari et al., 2007; Pei et al., 2010; Strauss et al., 2011). The integration time step ( $\Delta t$ ), is taken to be proportional to  $r^2$  ( $r$  is the distance from the Sun) avoiding oversampling in the outer heliosphere and therefore saving CPU time (Alanko-Huotari et al., 2007). We considered the 2D (radius and polar angle) approximation of Eq. (1) (Potgieter et al., 1993), and from this we calculate the equivalent set of SDEs (Bobik et al., 2011):

$$\begin{aligned} \Delta r &= \frac{1}{r^2} \frac{\partial(r^2 K_{rr})}{\partial r} \Delta t + (v_{sw} + v_{D_r} + v_{D_{NS}}) \cdot \Delta t + R_g \sqrt{2K_{rr} \Delta t} \\ \Delta \mu &= \frac{1}{r^2} \frac{\partial[(1 - \mu^2) K_{\theta\theta}]}{\partial \mu} \Delta t - \frac{\sqrt{1 - \mu^2}}{r} v_{D_\theta} \Delta t + R_g \sqrt{\frac{2K_{\theta\theta}(1 - \mu^2) \Delta t}{r^2}} \\ \Delta T &= -\left(\frac{2}{3} \frac{\rho V_{sw} T}{r}\right) \Delta t \end{aligned} \quad (2-4)$$

where  $\mu = \cos \theta$ , with  $\theta$  polar angle, and  $R_g$  is a gaussian distributed random number with unitary variance. Here the particle drift velocity is splitted in regular drift (radial drift  $v_{D_r}$ , latitudinal drift  $v_{D_\theta}$ ) and neutral sheet drift ( $v_{D_{NS}}$ ) as described by Potgieter and Moraal (1985) and Hatting and Burger (1995). The diffusion tensor is taken to be  $K_{rr} = K_{\parallel} \cos^2 \psi + K_{\perp} r \sin^2 \psi$  and  $K_{\theta\theta} = K_{\perp \theta}$  (Potgieter et al., 1993; Potgieter and Le Roux, 1994), where  $\psi$  is the angle between radial and magnetic field directions; labels  $\perp$  and  $\parallel$  are respectively the perpendicular and parallel

components of the diffusion process with respect to the background magnetic field lines. In heliocentric spherical coordinates, the perpendicular diffusion coefficient has two components, one along the radial direction,  $K_{\perp r}$ , the other one along the polar direction  $K_{\perp \theta}$ .  $\rho_k$  is the ratio between  $K_{\perp r}$  and the parallel diffusion coefficient  $K_{\parallel}$ :

$$K_{\perp r} = \rho_k K_{\parallel} \quad (5)$$

In the present model, we use  $\rho_k = 0.05$ : this value is in the mid of the range suggested by Palmer (1982) – see also Giacalone (1998) and Section 6.3 of Burger et al. (2000). The value of the perpendicular diffusion coefficient in the polar direction ( $K_{\perp \theta}$ ) can be assumed to be equal to the perpendicular diffusion coefficient in the radial direction, but we also consider an enhancement factor of  $\sim 10$  in the polar regions (see Potgieter, 2000), as described in Bobik et al. (2011, 2012). The parallel diffusion coefficient is  $K_{\parallel} = k_0 \beta K_P (P)(B_{\oplus}/3B)$  (Potgieter and Le Roux, 1994): here  $k_0 \approx 0.05 - 0.3 \times 10^{-3} \text{ AU}^2 \text{ GV}^{-1} \text{ s}^{-1}$ , is a diffusion parameter depending on solar activity (see Section 2.3),  $\beta$  is the particle velocity in unit of light speed  $c$ . We are interested to an interval of energy above 1 GeV where  $K_P = P$  (Potgieter and Le Roux, 1994), with  $P = pc/Ze$  is the CR particle's rigidity,  $p$  is the particle's momentum and  $Ze$  is the particle's charge.  $B_{\oplus}$  is the value of heliospheric magnetic field measured at the Earth orbit, and  $B$  is the magnitude of the heliospheric magnetic field (HMF) (Hatting and Burger, 1995):

$$B = \frac{A}{r^2} (e_r - \Gamma e_\phi) \cdot [1 - 2H(\theta - \theta')] \quad (6)$$

where  $A$  is a coefficient that allows  $|B|$  to be equal to  $B_{\oplus}$ , i.e., the value of HMF at the Earth orbit, and determines the field polarity, i.e.,  $A > 0$  for positive periods (e.g. AMS-01 observations) and  $A < 0$  for negative periods (e.g. PAMELA observations);  $\theta'$  is the polar angle determining the position of the heliospheric current sheet (HCS) (Jokipii and Thomas, 1981);  $H$  is the Heaviside function, thus  $[1 - 2H(\theta - \theta')]$  accounts for the change of sign between the two regions - above and below the HCS - of the heliosphere; finally  $\Gamma = \tan \Psi \cong \frac{\omega r \sin \Theta}{v_{sw}}$ , with  $\psi$  the spiral angle. We modify the HMF according to Jokipii and Kóta (1989), increasing the magnitude of the HMF in the polar regions (see Bobik et al., 2011 for details). We use a SW broad smoothed profile according to Ulysses observation for periods of low solar activity (McComas et al., 2000, 2008), described by the relation  $V_{sw}(\theta) = V_{max}$  if  $\theta \leq 30^\circ$  or  $\theta \geq 150^\circ$  and  $V_{sw}(\theta) = V_0 \cdot (1 + |\cos \theta|)$  if  $30^\circ < \theta < 150^\circ$  where  $V_0$  is approximately 400 km/s and  $V_{max}$  is 760 km/s.

### 2.2. Particle drift

We emphasize the importance to include particle drift in the model, since this is the only part of Eq. (1) that is sensitive to particle charge sign (or, equivalently, to the polarity of HMF). The particle drift is described by the relation (Jokipii and Levy, 1977):

$$v_{dr} = \nabla \times (K_T \mathbf{e}_B) \quad (7)$$

Here  $\mathbf{e}_B$  is a unit vector in the direction of HMF,  $K_T = \frac{pv}{3qB}$  is the antisymmetric part of the diffusion tensor with  $p$  particle momentum,  $v$  speed,  $q = Ze$  charge and  $B$  the magnitude of the magnetic field. The heliosphere is divided in two hemisphere of opposite HMF polarity. The transition region, called neutral sheet (NS), characterized by having  $|\mathbf{B}| = 0$ , swings inside a tilt angle ( $\alpha$ ), which value depends on the solar activity. Particles crossing the neutral sheet experience an additional drift caused by the different orientation of the magnetic field. Therefore particle drift has two components, one related to large scale gradient and curvature of the field and one related to the presence of the neutral sheet. In 2D-approximation the neutral sheet becomes a region where both components of drift contribute to the particle transport and drift velocity could be expressed as  $v_d = v_{dr} + v_{NS}$ . To describe the neutral sheet drift we adopt the description by Potgieter and Moraal (1985): here the drift coefficient  $K_T$  is scaled by a transition function  $f(\theta)$  that simulates the effect of a wavy neutral sheet. The transition function sets the rate at which the regular drift coefficient goes to 0 on the ecliptic plane ( $\theta = \pi/2$ ) (Potgieter and Moraal, 1985) and, in this description, is related to  $\alpha$  (e.g., see Equation (23) of Burger and Potgieter, 1989).

### 2.3. Parameters estimation

The model depends on measured values of SW speed on the ecliptic plane ( $V_0$ ), tilt angle ( $\alpha$ ) of the neutral sheet and on estimated values of the diffusion parameter ( $k_0$ ). Values of the tilt angle  $\alpha$  come from the Wilcox Solar Laboratory and are computed using two different models, as described in Hoeksema (1995). In this work we use the one referred as “L” model. Although Ferreira and Potgieter (2003) have shown that “R” model in Hoeksema (1995) is better suited than the “L” model as a proxy for solar activity for periods when  $\alpha$  is increasing, using our Monte Carlo we found that observations are better reproduced using “L” model in any solar activity condition. Values of  $V_0$  and  $B_\oplus$  were obtained from NSSDC OMNIWeb system (<http://omniweb.gsfc.nasa.gov/form/dx1.html>) by 27 daily averages. We estimate the values of  $k_0$  following the procedure described in Bobik et al. (2011), where the diffusion parameter is fitted with a practical relationship between  $k_0$  and the monthly Smoothed Sunspot Number (SSN), taken from <http://www.sidc.oma.be/sunspot-data/>, which is used as solar activity monitor.

Our code simulates a diffusive propagation of CR particle entering the heliosphere from the termination shock, that we located at 100 AU (note that in Decker et al. (2007) the termination shock is located at 94 AU), and reaching the Earth. We evaluated the time  $t_{sw}$  needed by the SW to expand from the outer corona up to 100 AU: with a minimum speed of  $\approx 400$  km/s it takes nearly 14 months, while the time interval  $\tau_{ev}$  of the stochastic evo-

lution of a “quasi-particle” inside the heliosphere from 100 AU down to Earth is between 1 month (at 200 MeV) and few days (at 10 GeV). Therefore we have  $\tau_{ev} < t_{sw}$  and  $t_{sw} \gg 1$  month. In this scenario we should use monthly averages of the parameters with values evolving in time to describe the conditions of the heliosphere in the modulation process. In fact at 100 AU, where particles are injected, the solar conditions are similar to those present at the Earth  $\sim 14$  months in advance. Therefore we split the heliosphere in 14 radial regions, equally spaced. In each region we used values of  $k_0$ ,  $\alpha$  and  $V_{sw}$ , evaluated at the time when the solar wind, present in that region, has been ejected by the sun. In this first approximation the latitudinal dependence has non been accounted for.

We also tested the effects of compressing the spherical heliosphere down to 80 AU or stretching it up to 120 AU. We found that the differences in the modulated flux are not relevant in the energy range of this work ( $\sim 1 - 10$  GeV) (see also Bobik et al., 2012). This is an indication that, at these energies, most of the modulation occurs in the inner heliosphere and that the structures beyond the termination shock are not considerably contributing, as shown by Bobik et al. (2008). This is in agreement with the results of Scherer et al. (2011) showing as modulation occurring in the heliosheath does not affect significantly particles with energy larger than a few hundreds of MeV.

A further improvement in the description of the heliosphere, which can be implemented in and investigated using the current code, is an aspherical solar cavity, related to the solar wind speed as a function of the polar angle  $\theta$ . (see e.g. Haasbroek and Potgieter, 1997; Langner and Potgieter, 2005)

### 2.4. Local interstellar spectrum

The LIS of electrons is mainly due to the primary component of particles diffusing in the galaxy. The LIS of positrons is, on the contrary, a secondary production due to GCR interactions with the interstellar medium. We used the model proposed by Zhang and Cheng (2001) (here expressed in unit of  $\text{GeV}^{-1} \text{s}^{-1} \text{sr}^{-1} \text{m}^{-2}$ ) based on calculation by Moskalenko and Strong (1998):

$$\phi_{e^-} = \frac{1600 \cdot T^{-1.1}}{1 + 11 \cdot T^{0.9} + 1.92 \cdot T^{2.45}} \quad (8)$$

$$\phi_{e^+} = \frac{45000 \cdot T^{0.7}}{1 + 650 \cdot T^{2.3} + 1500 \cdot T^{4.2}} \quad (9)$$

We corrected the lower-right term in Eq (8), in order to fit AMS-01 observations at energy  $> 20$  GeV, but we leave unmodified the low energy terms. Recent measurements of AMS-01 and PAMELA have shown an excess in the cosmic ray positron fraction, which cannot be accounted for by the usual galactic production models. Here we include in the positron LIS an additional term like the one proposed by Grasso et al. (2009), that simulates the

possible contribution of local Pulsars. This term helps to adapt LIS to observations at energy  $>10$  GeV, but does not affect modulated spectra at lower energy.

### 3. Results

#### 3.1. Particle drift effect on modulation

The charge sign dependence of GCR propagation in the heliosphere is due to the particle drift term of the Parker equation. The fundamental parameter setting the direction of particle drift is  $qA$ , where  $q$  is the particle charge and  $A$  is a coefficient that accounts for the field polarity. We made a simulation of propagation of electrons and positrons during a solar minimum, reproducing the solar activity conditions occurred in June 1998, assuming both the solar field polarities. We found, in agreement with theory, that the flux for  $qA < 0$  is systematically lower than the flux for  $qA > 0$  (see e.g. Potgieter and Langner, 2004; Alanko-Huotari et al., 2007, for similar results). Results are shown in Fig. 1, where positron and electron spectra are compared. In Fig. 2 we show the modulated cosmic ray positron fraction above 1 GeV: for  $A > 0$  modulated positron fraction is comparable with the LIS positron fraction; while for  $A < 0$  a reduction of the cosmic ray positron fraction is expected and it is even more relevant going to low kinetic energy.

#### 3.2. Measurements with AMS-01 and PAMELA

We selected two experiments in similar solar conditions but opposite magnetic field polarity. AMS-01 (Aguilar et al., 2007; Alcaraz et al., 2000) operated on board of the Space Shuttle in June 1998, at the end of a solar minimum occurred during a period with  $A > 0$ . PAMELA (Adriani et al., 2009) is a space born experiment working since July 2006. Published data have been taken between 2006

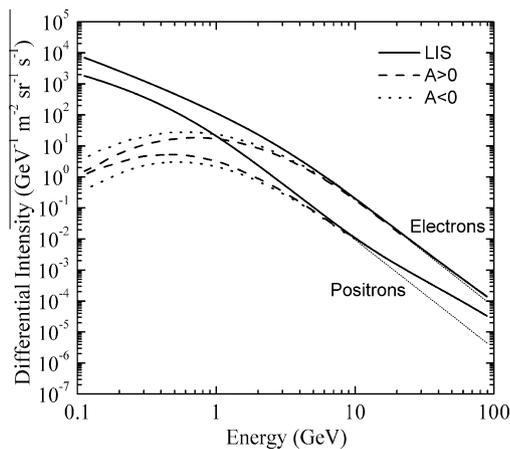


Fig. 1.  $e^-$  and  $e^+$  modulation for a period corresponding to a typical solar minimum (we used the conditions occurred in June 1998) with both magnetic field polarities. Short-dotted lines at high energy represent the two LIS due to only Eqs. (8) and (9).

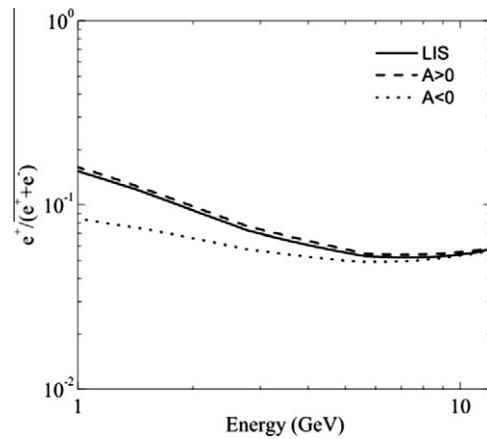


Fig. 2. The cosmic ray positron fraction evaluated for a period corresponding to a typical solar minimum with both magnetic field polarities. We used the results shown in Fig. 1.

and December 2008, therefore during the last long solar minimum with  $A < 0$ .

We mainly reproduce, within the error bars, the AMS-01 observations for electrons and positrons (see Figs. 3 and 4). We find a good agreement between simulations and observations also for the cosmic ray positron fraction. Despite the large solar modulation of both electrons and positrons, for AMS-01 the modulated ratio is very close to the interstellar ratio (see Fig. 5), as obtained in previous results for  $A > 0$ . We used the same model of propagation to reproduce PAMELA observations for the period 2006–2008. Due to the wide time interval covered by the analysis, we performed several simulations using parameters related to the full period of observations and taking their average value. In Fig. 5 it is shown that, for PAMELA, the modulated cosmic ray positron fraction is lower than the interstellar one, as expected.

#### 3.3. Predictions for AMS-02

Our simulation code has been also used to predict GCR spectra for future observations. The periodic behavior of

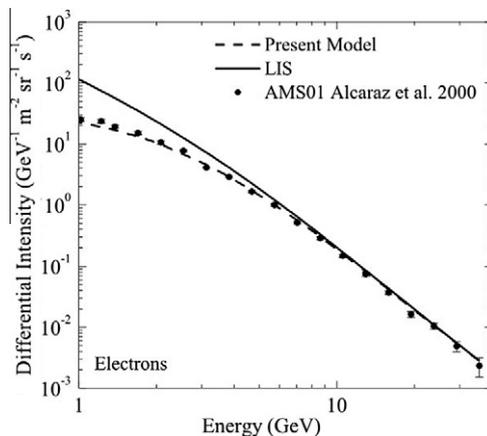


Fig. 3. Simulated electron spectrum for AMS-01 mission (1998).

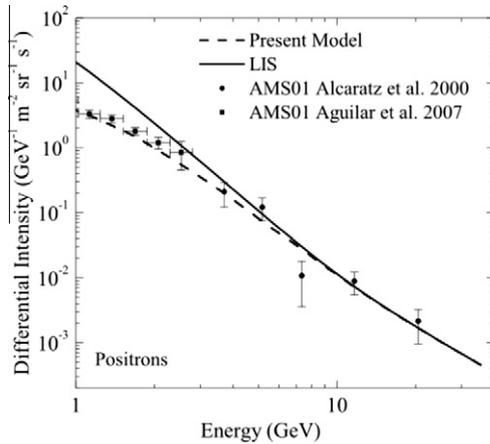


Fig. 4. Simulated positron spectrum for AMS-01 mission (1998).

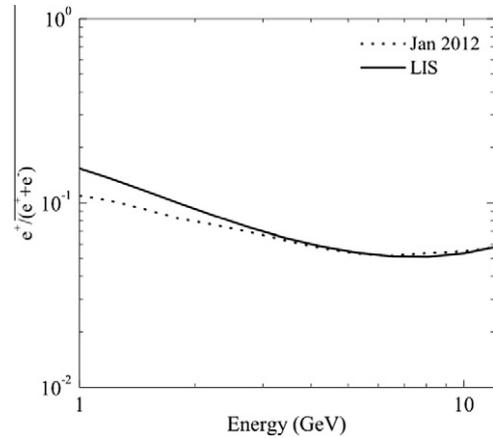


Fig. 6. Prediction of the cosmic ray positron fraction for January 2012.

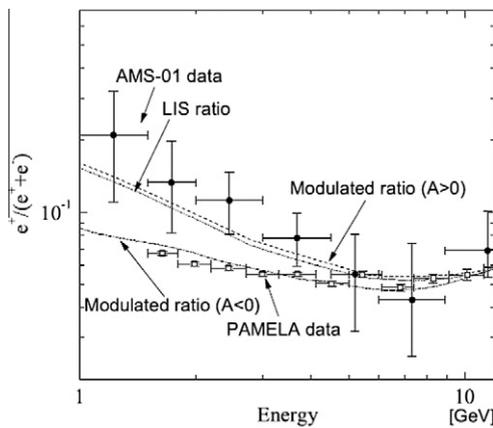


Fig. 5. Simulated cosmic ray positron fraction for AMS-01 (1998) and PAMELA (2006–2008).

the heliosphere allows us to predict, with a certain level of confidence, the value of solar modulation parameters to be used in the simulation. The assumption is that diffusion coefficient, tilt angle and solar wind speed show a near-regular and almost periodic trend. The periodicity takes two consecutive 11-years solar cycles. In order to get these parameters we considered the prediction of Smoothed Sunspot Number from IPS (Ionospheric Prediction Service) of the Australian Bureau of Meteorology. Using SIDAC data (Solar Influences Data Analysis Center) we selected periods, in the past, with similar solar activity conditions and same solar field polarity of the simulation time: therefore approximately 22 years in advance.

Here we present the simulations for AMS-02 mission, that has been installed on the ISS on May 19, 2011. Simulations are related to January 2012, when the mission will reach a stable configuration. It is important to note that anomalous non periodic features in solar activity or different values of SSN with respect the predicted ones, could falsify this forecast estimation. Results are shown in Fig. 6. At that time we still have  $A < 0$  and the modulated cosmic ray positron fraction is still below the interstellar one.

#### 4. Conclusions

We used a 2D stochastic Monte Carlo code for charged particle modulation in the heliosphere. Our model takes in to account particle drift effects. We focused our attention on the modulation of electrons and positrons. We show the different behavior occurring in periods with different magnetic field polarity for the two species. We find that during periods with  $A > 0$  (e.g. AMS-01 mission) the positrons ratio is quite similar to the interstellar model. During periods with opposite field polarity (e.g. PAMELA mission) we find instead a relevant reduction of the cosmic ray positron fraction at energy below 10 GeV in comparison with the interstellar one. This feature is even more relevant going to lower energy. We find also a quantitatively good agreement with the observed values (AMS-01 and PAMELA), for both positive and negative periods and for both particles. We conclude that observations can be well explained with a polarity-dependent effect of solar modulation, due to the particle drift. Although less evident, AMS-02, in the first period of its data taking, will probably observe a modulation effect similar to the one measured by PAMELA.

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