

Contents

1	Vectors and functions	1
1.A	Some vector algebra essentials	2
1.B	Introduction to sets	9
1.C	Real-valued functions	17
1.D	Coordinate systems	25
1.E	Drawing or visualizing surfaces in \mathbb{R}^3	27
1.F	Level sets	38
1.G	Supplementary problems	43
2	Differentiation of multivariable functions	49
2.A	The derivative	49
2.B	Limits and continuity	53
2.C	Partial derivatives	62
2.D	Differentiability of $f : \mathbb{R}^n \rightarrow \mathbb{R}$	67
2.E	Directional derivatives and the gradient	74
2.F	Higher-order derivatives	80
2.G	Composite functions and the chain rule	84
2.H	Implicit functions	101
2.I	Taylor's formula and Taylor series	113
2.J	Supplementary problems	119
3	Applications of the differential calculus	125
3.A	Extreme values of $f : \mathbb{R}^n \rightarrow \mathbb{R}$	125

3.B	Extreme points: The complete story	133
3.C	Differentials and error analysis	145
3.D	Method of least squares	146
3.E	Partial derivatives in equations: Partial differential equations	152
3.F	Supplementary problems	171
4	Integration of multivariable functions	177
4.A	Multiple integrals	177
4.B	Iterated integration in \mathbb{R}^2	184
4.C	Integration over complex domains	187
4.D	Generalized (improper) integrals in \mathbb{R}^2	193
4.E	Change of variables in \mathbb{R}^2	198
4.F	Triple integrals	204
4.G	Iterated integration in \mathbb{R}^3	207
4.H	Change of variables in \mathbb{R}^3	211
4.I	n -tuple integrals	213
4.J	Epilogue: Some practical tips for evaluating integrals	215
4.K	Supplementary problems	217
5	Vector calculus	223
5.A	Vector-valued functions	223
5.B	Vector fields	238
5.C	Line integrals	246
5.D	Surface integrals	260
5.E	Gauss's theorem	273
5.F	Green's and Stokes's theorems	281
5.G	Supplementary problems	293
	Glossary of symbols	301
	Bibliography	305
	Index	307