

## CONTENTS

FOREWORD . . . . .	xi
PREFACE . . . . .	xiv
APOLOGIA PRO LIBRO SUO . . . . .	xvii
HIGHLIGHTS & UNUSUAL FEATURES . . . . .	xviii
Chapter 0	
PREREQUISITES AND PRELIMINARIES	
§1 Set Theory . . . . .	1
§2 Algebra . . . . .	3
§3 The Battlefield . . . . .	3
§4 Metric Spaces . . . . .	11
§5 Limsup and All That . . . . .	15
§6 Continuous Functions . . . . .	17
§7 Calculus . . . . .	19
§8 <i>De Figuris</i> . . . . .	20
§9 On Problems and Exercises . . . . .	21
Chapter I	
CURVES, CONNECTEDNESS AND CONVEXITY	
§1 Elementary Results on Connectedness . . . . .	23
§2 Connectedness of Intervals, Curves, and Convex Sets . . . . .	25
§3 The Basic Connectedness Lemma . . . . .	34
§4 Components and Compact Exhaustions . . . . .	35
§5 Connectivity of a Set . . . . .	40
§6 Extension Theorems . . . . .	44
Notes to Chapter I . . . . .	49
Chapter II	
(COMPLEX) DERIVATIVE AND (CURVILINEAR) INTEGRALS	
§1 Holomorphic and Harmonic Functions . . . . .	53
§2 Integrals along Curves . . . . .	58
§3 Differentiating under the Integral Sign . . . . .	66

§4 Elementary Real-variables Methods . . . . .	69
§5 Complex Logarithms, <i>avant la lettre</i> . . . . .	84
Notes to Chapter II . . . . .	90
Chapter III	
POWER SERIES AND THE EXPONENTIAL FUNCTION	
§1 Introduction . . . . .	97
§2 Power Series . . . . .	98
§3 The Complex Exponential Function . . . . .	110
§4 Bernoulli Polynomials, Numbers and Functions . . . . .	154
§5 Cauchy's Integral Formula Adumbrated . . . . .	160
§6 Holomorphic Logarithms Previewed . . . . .	162
Notes to Chapter III . . . . .	176
Chapter IV	
THE INDEX AND SOME PLANE TOPOLOGY	
§1 Introduction . . . . .	187
§2 Curves Winding around Points . . . . .	187
§3 Homotopy and the Index . . . . .	196
§4 Existence of Continuous Logarithms . . . . .	200
§5 The Jordan Curve Theorem . . . . .	220
§6 Applications of the Foregoing Technology . . . . .	227
§7 Continuous and Holomorphic Logarithms in Open Sets . . . . .	242
§8 Simple Connectivity for Open Sets . . . . .	247
Notes to Chapter IV . . . . .	250
Chapter V	
CONSEQUENCES OF THE CAUCHY-GOURSAT THEOREM—	
MAXIMUM PRINCIPLES AND THE LOCAL THEORY	
§1 Goursat's Lemma and Cauchy's Theorem for Starlike Regions . . . . .	261
§2 Maximum Principles . . . . .	271
§3 The Dirichlet Problem for Disks . . . . .	283
§4 Existence of Power Series Expansions, Cauchy's Formula, Removable Singularities . . . . .	308
§5 Harmonic Majorization . . . . .	328
§6 Uniqueness Theorems . . . . .	347

§7 Local Theory . . . . .	361
Notes to Chapter V . . . . .	382
Chapter VI	
SCHWARZ' LEMMA AND ITS MANY APPLICATIONS	
§1 Schwarz' Lemma and the Conformal Automorphisms of Disks . . . . .	397
§2 Boundary-preserving Maps . . . . .	413
§3 Applications to Half-planes, Strips and Annuli . . . . .	421
§4 The Theorem of Carathéodory, Julia, Wolff, <i>et al.</i> . . . . .	427
§5 Subordination . . . . .	440
Notes to Chapter VI . . . . .	450
Chapter VII	
CONVERGENT SEQUENCES OF HOLOMORPHIC FUNCTIONS	
§1 Convergence in $H(U)$ . . . . .	457
§2 Applications of the Convergence Theorems; Boundedness Criteria and Covering Results . . . . .	475
§3 Prescribing Zeros . . . . .	490
§4 Elementary Iteration Theory . . . . .	499
§5 Iteration in Disks and Annuli . . . . .	508
Notes to Chapter VII . . . . .	517
Chapter VIII	
POLYNOMIAL AND RATIONAL APPROXIMATION— RUNGE THEORY	
§1 The Basic Integral Representation Theorem . . . . .	529
§2 Applications to Approximation . . . . .	533
§3 Other Applications of the Integral Representation . . . . .	541
§4 Some Special Kinds of Approximation . . . . .	551
§5 Carleman's Approximation Theorem . . . . .	559
§6 Harmonic Functions in a Half-Plane . . . . .	562
Notes to Chapter VIII . . . . .	578
Chapter IX	
THE RIEMANN MAPPING THEOREM	
§1 Introduction . . . . .	585
§2 The Proof of Carathéodory and Koebe . . . . .	591

§3 Fejér and Riesz' Proof and the Uniqueness Theorem . . . . .	597
§4 Boundary Behavior for Jordan Regions . . . . .	599
§5 A Few Applications of the Osgood-Taylor-Carathéodory Theorem . . . . .	607
§6 More on Jordan Regions and Boundary Behavior . . . . .	615
§7 Harmonic Functions and the General Dirichlet Problem . . . . .	624
§8 The Dirichlet Problem and the Riemann Mapping Theorem . . . . .	638
§9 Conformal Mapping of a Half-plane onto a Polygon . . . . .	644
Notes to Chapter IX . . . . .	656

## Chapter X

## SIMPLE AND DOUBLE CONNECTIVITY

§1 Simple Connectivity . . . . .	667
§2 Double Connectivity . . . . .	679
§3 Higher Connectivity . . . . .	688
§4 The Aumann-Carathéodory Theorem . . . . .	695
§5 The Fundamental Mapping Theorem for Doubly-connected Regions Revisited . . . . .	699
Notes to Chapter X . . . . .	709

## Chapter XI

## ISOLATED SINGULARITIES

§1 Laurent Series and Classification of Singularities . . . . .	713
§2 Rational Functions . . . . .	727
§3 Isolated Singularities on the Circle of Convergence . . . . .	740
§4 The Residue Theorem and Some Applications . . . . .	742
§5 Specifying Principal Parts—Mittag-Leffler's Theorem . . . . .	777
§6 Meromorphic Functions . . . . .	786
§7 Poisson's Formula in an Annulus and Isolated Singularities of Harmonic Functions . . . . .	804
Notes to Chapter XI . . . . .	818

## Chapter XII

## OMITTED VALUES AND NORMAL FAMILIES

§1 Logarithmic Means and Jensen's Inequality . . . . .	829
§2 Miranda's Theorem . . . . .	836
§3 Immediate Applications of Miranda . . . . .	852

§4 Normal Families and Julia's Extension of Picard's Great Theorem . . . . .	858
§5 Sectorial Limit Theorems . . . . .	865
§6 Applications to Iteration Theory . . . . .	876
§7 Ostrowski's Proof of Schottky's Theorem and an Application . . . . .	880
Notes to Chapter XII . . . . .	890

BIBLIOGRAPHY . . . . .	899
------------------------	-----

NAME INDEX . . . . .	1076
----------------------	------

SUBJECT INDEX . . . . .	1100
-------------------------	------

SYMBOL INDEX . . . . .	1118
------------------------	------

SERIES SUMMED, INTEGRALS EVALUATED . . . . .	1121
--	------