

Contents

1	Introduction	1
1.1	Basic Properties and Examples	1
1.2	Universality of the Hilbert Cube	3
1.3	Decreasing Sequences of Continua	4
1.4	Advanced Examples	5
1.5	Decompositions	14
1.6	The Brouwer Reduction Theorem	15
1.7	Exercises	16
2	Locally Connected Continua	21
2.1	Property S	22
2.2	An Auxiliary Mapping	23
2.3	The Hahn–Mazurkiewicz Theorem	27
2.4	Exercises	29
3	Cutting Wires and Bumping Boundaries	33
3.1	The Cut Wire Theorem	33
3.2	The Boundary Bumping Theorem	34
3.3	Exercises	36
4	Indecomposable Continua	39
4.1	Composants	39
4.2	Another Characterization	42
4.3	Exercises	43
5	Characterizing Arcs and Circles	45
5.1	Existence of Non-cut Points	45
5.2	Arcs	46
5.3	Simple Closed Curves	48
5.4	Exercises	50

6 Finite Graphs	53
6.1 Definition and Characterizations	53
6.2 Order of a Subset.....	55
6.3 Exercises	59
7 Dendroids.....	63
7.1 Definition and Problem	63
7.2 Maximal Arcs	64
7.3 Dendrites	66
7.4 Semi-combs	72
7.5 Exercises	76
8 The Cantor Set.....	79
8.1 The Cantor Set as a Product	79
8.2 Images of the Cantor Set	82
8.3 A Characterization	82
8.4 Two Important Mappings.....	86
8.5 Exercises	86
9 Hyperspaces of Continua	89
9.1 The Hausdorff Metric	91
9.2 Compactness	93
9.3 Whitney Mappings.....	98
9.4 Order Arcs and Connectedness.....	99
9.5 Whitney Levels.....	102
9.6 Exercises	105
10 Models of Hyperspaces.....	113
10.1 $C([0, 1])$	113
10.2 $C(S^1)$	115
10.3 $C(\text{Simple Triod})$	115
10.4 $C(\text{Noose})$	116
10.5 No More Peano Models of $C(X)$ in \mathbb{R}^3	117
10.6 More Continua X for Which $C(X)$ is Embeddable in \mathbb{R}^3	119
10.7 Peano X for Which $C(X)$ is Embeddable in \mathbb{R}^4 and \mathbb{R}^5	121
10.8 Infinite-Dimensional Models of $C_n(X)$	123
10.9 $C_n([0, 1])$ for $n \geq 2$	123
10.10 $C_n(S^1)$ for $n \geq 2$	124
10.11 Continua for Which $C(X)$ is a Cone	124
10.12 Models of 2^X	125
10.13 $F_n([0, 1])$	125
10.14 $F_n(S^1)$	127
10.15 $F_2(\text{Simple Triod})$	129
10.16 $F_2(\text{Simple 4-od})$	130
10.17 $F_2(\text{Noose})$	130
10.18 $F_2(\text{Figure Eight Continuum})$	132

10.19	Hyperspaces $C_n(X)/F_m(X)$, $m \leq n$; and $F_n(X)/F_m(X)$, $m < n$	133
10.20	$F_2(\sin(\frac{1}{x}))$ -Continuum.....	134
10.21	More Questions	134
10.22	F_n (Hilbert Cube).....	135
11	Irreducible Continua	137
11.1	Irreducibility	137
11.2	Closed Domains.....	138
11.3	Main Theorem	143
11.4	Exercises	145
12	Unicoherence	147
12.1	Unicoherence and Property (b).....	147
12.2	Open Unicoherence	153
12.3	The Disk.....	158
12.4	The Mountain Climbing Theorem	158
12.5	The Fundamental Theorem of Algebra	160
12.6	Exercises	162
13	The Fixed Point Property	165
13.1	Introduction	165
13.2	Dog Chasing Rabbit	166
13.3	Cells	167
13.4	Dendroids	167
13.5	The Cone of a Spiral	168
13.6	Exercises	171
14	Inverse Limits	175
14.1	Definition and Examples	175
14.2	Indecomposability	177
14.3	The Anderson–Choquet Theorem.....	178
14.4	Chainable Continua as Inverse Limits	179
14.5	Generalized Inverse Limits	185
14.6	Exercises	186
15	Homogeneity of the Hilbert Cube	191
15.1	Introduction	191
15.2	The Proof	191
15.3	Exercises	196
16	Absolute Retracts	197
16.1	General Theory.....	197
16.2	A Characterization	202
16.3	Exercises	204

17 Stronger Properties of the Pseudo-Arc	207
17.1 Chains	208
17.2 Terminal and Final Points	209
17.3 An Auxiliary Result.....	216
17.4 Patterns	217
17.5 Stronger Properties of the Pseudo-Arc.....	223
17.6 Exercises	228
References.....	229
Index.....	235