

Contents

Preface	ix
1 Introduction to Ordinary Differential Equations	1
1.1 Existence and Uniqueness	1
1.2 Types of Differential Equations	6
1.3 Geometric Interpretation of Autonomous Systems	8
1.4 Flows	14
1.5 Stability and Linearization	18
1.6 Stability and the Direct Method of Lyapunov	25
1.7 Manifolds	31
1.7.1 Introduction to Invariant Manifolds	32
1.7.2 Smooth Manifolds	41
1.7.3 Tangent Spaces	50
1.7.4 Change of Coordinates	58
1.7.5 Reparametrization of Time	63
1.7.6 Polar Coordinates	66
1.8 Periodic Solutions	82
1.8.1 The Poincaré Map	83
1.8.2 Limit Sets and Poincaré–Bendixson Theory	92
1.9 Regular and Singular Perturbation	109
1.10 Review of Calculus	113
1.10.1 The Mean Value Theorem	118
1.10.2 Integration in Banach Spaces	120

1.11	Contraction	126
1.11.1	The Contraction Mapping Theorem	127
1.11.2	Uniform Contraction	128
1.11.3	Fiber Contraction	132
1.11.4	The Implicit Function Theorem	139
1.12	Existence, Uniqueness, and Extension	140
2	Homogeneous Linear Systems	151
2.1	Gronwall's Inequality	152
2.2	Existence Theory	153
2.3	Principle of Superposition	155
2.4	Linear Equations with Constant Coefficients	160
2.5	The Matrix Exponential	166
2.6	Lie–Trotter and Baker–Campbell–Hausdorff formulas	177
3	Stability of Linear Systems	183
4	Stability of Nonlinear Systems	189
4.1	Variation of Parameters and Solution of Inhomogeneous Linear Systems	189
4.2	Alekseev–Gröbner formula	193
4.3	Stability of Nonlinear Systems	196
4.4	An Instability Criterion	202
5	Floquet Theory	207
5.1	Lyapunov Exponents	222
5.2	Hill's Equation	225
5.3	Periodic Orbits of Linear Systems	229
5.4	Stability of Periodic Orbits	231
6	Applications	245
6.1	Origins of ODE: Calculus of Variations	245
6.2	Origins of ODE: Classical Physics	260
6.2.1	Motion of a Charged Particle	263
6.2.2	Motion of a Binary System	264
6.2.3	Perturbed Kepler Motion and Delaunay Elements	273
6.2.4	Satellite Orbiting an Oblate Planet	280
6.2.5	The Diamagnetic Kepler Problem	286
6.3	Coupled Pendula: Normal Modes and Beats	292
6.4	The Fermi–Ulam–Pasta Oscillator	296
6.5	The Inverted Pendulum	301
6.6	Origins of ODE: Partial Differential Equations	307
6.6.1	Infinite-Dimensional ODE	309
6.6.2	Galérkin Approximation	322

6.6.3	Traveling Waves	334
6.6.4	First Order PDE	339
6.7	Control	346
6.7.1	Controllability of Time-Invariant Linear Systems	347
6.7.2	Optimal Control	363
6.7.3	Quadratic Regulator	371
6.7.4	Optimal Control Example	375
6.7.5	Parameter Estimation and the Adjoint Method	385
7	Hyperbolic Theory	395
7.1	Invariant Manifolds	395
7.2	Applications of Invariant Manifolds	419
7.3	The Hartman–Grobman Theorem	422
7.3.1	Diffeomorphisms	423
7.3.2	Differential Equations	429
7.3.3	Linearization via the Lie Derivative	434
8	Continuation of Periodic Solutions	445
8.1	A Classic Example: van der Pol’s Oscillator	446
8.1.1	Continuation Theory and Applied Mathematics	452
8.2	Autonomous Perturbations	454
8.2.1	Poincaré’s Method of Continuation	455
8.2.2	Continuation of Periodic Orbits of Planar Systems	457
8.2.3	Diliberto’s Theorem	458
8.2.4	Preparation Theorem and Persistence of Nonhyperbolic Periodic Orbits	463
8.2.5	Continuation from an Annulus of Period Orbits	467
8.2.6	Periodic Orbits of Multidimensional Systems with First Integrals	470
8.3	Nonautonomous Perturbations	477
8.3.1	Rest Points	480
8.3.2	Isochronous Period Annulus	481
8.3.3	The Forced van der Pol Oscillator	485
8.3.4	Regular Period Annulus and Lyapunov–Schmidt Reduction	493
8.3.5	Limit Cycles–Entrainment–Resonance Zones	504
8.3.6	Lindstedt Series and the Perihelion of Mercury	511

8.3.7	Entrainment Domains for van der Pol's Oscillator	520
8.4	Forced Oscillators	522
9	Homoclinic Orbits, Melnikov's Method, and Chaos	529
9.1	Autonomous Perturbations: Separatrix Splitting	535
9.2	Periodic Perturbations: Transverse Homoclinic Points	545
9.3	Origins of ODE: Fluid Dynamics	559
9.3.1	The Equations of Fluid Motion	560
9.3.2	ABC Flows	570
9.3.3	Chaotic ABC Flows	573
10	Averaging	591
10.1	The Averaging Principle	591
10.2	Averaging at Resonance	601
10.3	Action-Angle Variables	618
11	Bifurcation	623
11.1	One-Dimensional State Space	624
11.1.1	The Saddle-Node Bifurcation	624
11.1.2	A Normal Form	626
11.1.3	Bifurcation in Applied Mathematics	627
11.1.4	Families, Transversality, and Jets	629
11.2	Saddle-Node Bifurcation via Lyapunov-Schmidt Reduction	637
11.3	Poincaré-Andronov-Hopf Bifurcation	643
11.3.1	Multiple Hopf Bifurcation	655
11.4	Dynamic Bifurcation	674
11.5	Global Continuation and the Crandall-Rabinowitz Theorem	682
11.5.1	Finite-Dimensional Approximation	683
11.5.2	Continuation	684
11.5.3	Bifurcation	687
11.5.4	Summary	689
	References	693
	Index	711