

## TABLE OF CONTENTS

Preface for the Second Edition . . . . .		vi
Preface . . . . .		viii
Contents, Volume II . . . . .		xi
I. The Complex Number System		
§1. The real numbers . . . . .		1
§2. The field of complex numbers . . . . .		1
§3. The complex plane . . . . .		3
§4. Polar representation and roots of complex numbers . . . . .		4
§5. Lines and half planes in the complex plane . . . . .		6
§6. The extended plane and its spherical representation . . . . .		8
II. Metric Spaces and the Topology of $\mathbb{C}$		
§1. Definition and examples of metric spaces . . . . .		11
§2. Connectedness . . . . .		14
§3. Sequences and completeness . . . . .		17
§4. Compactness . . . . .		20
§5. Continuity . . . . .		24
§6. Uniform convergence . . . . .		28
III. Elementary Properties and Examples of Analytic Functions		
§1. Power series . . . . .		30
§2. Analytic functions . . . . .		33
§3. Analytic functions as mapping, Möbius transformations . . . . .		44
IV. Complex Integration		
§1. Riemann-Stieltjes integrals . . . . .		58
§2. Power series representation of analytic functions . . . . .		68
§3. Zeros of an analytic function . . . . .		76
§4. The index of a closed curve . . . . .		80
§5. Cauchy's Theorem and Integral Formula . . . . .		83
§6. The homotopic version of Cauchy's Theorem and simple connectivity . . . . .		87
§7. Counting zeros; the Open Mapping Theorem . . . . .		97
§8. Goursat's Theorem . . . . .		100
V. Singularities		
§1. Classification of singularities . . . . .		103

§2.	Residues	112
§3.	The Argument Principle	123
VI.	The Maximum Modulus Theorem	
§1.	The Maximum Principle	128
§2.	Schwarz's Lemma	130
§3.	Convex functions and Hadamard's Three Circles Theorem	133
§4.	Phragmén-Lindelöf Theorem	138
VII.	Compactness and Convergence in the Space of Analytic Functions	
§1.	The space of continuous functions $C(G, \Omega)$	142
§2.	Spaces of analytic functions	151
§3.	Spaces of meromorphic functions	155
§4.	The Riemann Mapping Theorem	160
§5.	Weierstrass Factorization Theorem	164
§6.	Factorization of the sine function	174
§7.	The gamma function	176
§8.	The Riemann zeta function	187
VIII.	Runge's Theorem	
§1.	Runge's Theorem	195
§2.	Simple connectedness	202
§3.	Mittag-Leffler's Theorem	204
IX.	Analytic Continuation and Riemann Surfaces	
§1.	Schwarz Reflection Principle	210
§2.	Analytic Continuation Along A Path	213
§3.	Monodromy Theorem	217
§4.	Topological Spaces and Neighborhood Systems	221
§5.	The Sheaf of Germs of Analytic Functions on an Open Set	227
§6.	Analytic Manifolds	233
§7.	Covering spaces	245
X.	Harmonic Functions	
§1.	Basic Properties of harmonic functions	252
§2.	Harmonic functions on a disk	256
§3.	Subharmonic and superharmonic functions	263
§4.	The Dirichlet Problem	269
§5.	Green's Functions	275

Table of Contents	xi
<b>XI. Entire Functions</b>	
§1. Jensen's Formula	280
§2. The genus and order of an entire function	282
§3. Hadamard Factorization Theorem	287
<b>XII. The Range of an Analytic Function</b>	
§1. Bloch's Theorem	292
§2. The Little Picard Theorem	296
§3. Schottky's Theorem	297
§4. The Great Picard Theorem	300
Appendix A: Calculus for Complex Valued Functions on an Interval	303
Appendix B: Suggestions for Further Study and Bibliographical Notes	307
References	311
Index	313
List of Symbols	317

## TABLE OF CONTENTS, VOLUME II

### Preface

#### 13. Return to Basics

- §1. Regions and Curves
- §2. Derivatives and other recollections
- §3. Harmonic conjugates and primitives
- §4. Analytic arcs and the reflection principle
- §5. Boundary values for bounded analytic functions

#### 14. Conformal Equivalence For Simply Connected Regions

- §1. Elementary properties and examples
- §2. Crosscuts
- §3. Prime Ends
- §4. Impressions of a prime end
- §5. Boundary values of Riemann maps

- §6. The Area Theorem
- §7. Disk mappings: the class  $\mathcal{S}$
  
- 15. Conformal Equivalence For Finitely Connected Regions**
  - §1. Analysis on a finitely connected region
  - §2. Conformal equivalence with an analytic Jordan region
  - §3. Boundary values for a conformal equivalence between finitely connected Jordan regions
  - §4. Convergence of univalent functions
  - §5. Conformal equivalence with a circularly slit annulus
  - §6. Conformal equivalence with a circularly slit disk
  - §7. Conformal equivalence with a circular region
  
- 16. Analytic Covering Maps**
  - §1. Results for abstract covering spaces
  - §2. Analytic covering spaces
  - §3. The modular function
  - §4. Applications of the modular function
  - §5. The existence of the universal analytic covering map
  
- 17. De Branges's Proof of the Bieberbach Conjecture**
  - §1. Subordination
  - §2. Loewner chains
  - §3. Loewner's differential equation
  - §4. The Milin Conjecture
  - §5. Some special functions
  - §6. The proof of de Branges's Theorem
  
- 18. Some Fundamental Concepts From Analysis**
  - §1. Bergman spaces of analytic and harmonic functions
  - §2. Partitions of unity
  - §3. Convolution in Euclidean space
  - §4. Distributions
  - §5. The Cauchy transform
  - §6. An application: rational approximation
  - §7. Fourier series and Cesàro sums
  
- 19. Harmonic Functions Redux**
  - §1. Harmonic functions on the disk
  - §2. Fatou's Theorem
  - §3. Semicontinuous functions
  - §4. Subharmonic functions
  - §5. The logarithmic potential
  - §6. An application: approximation by harmonic functions
  - §7. The Dirichlet problem
  - §8. Harmonic majorants
  - §9. The Green function

- §10. Regular points for the Dirichlet problem
- §11. The Dirichlet principle and Sobolev spaces

## 20. Hardy Spaces on the Disk

- §1. Definitions and elementary properties
- §2. The Nevanlinna Class
- §3. Factorization of functions in the Nevanlinna class
- §4. The disk algebra
- §5. The invariant subspaces of  $H^p$
- §6. Szegő's Theorem

## 21. Potential Theory in the Plane

- §1. Harmonic measure
- §2. The sweep of a measure
- §3. The Robin constant
- §4. The Green potential
- §5. Polar sets
- §6. More on regular points
- §7. Logarithmic capacity: part 1
- §8. Some applications and examples of logarithmic capacity
- §9. Removable singularities for functions in the Bergman space
- §10. Logarithmic capacity: part 2
- §11. The transfinite diameter and logarithmic capacity
- §12. The refinement of a subharmonic function
- §13. The fine topology
- §14. Wiener's criterion for regular points

## References

## List of Symbols

## Index