

# Contents

Preface	vii
CHAPTER 1	
Topology of the Complex Plane and Holomorphic Functions	1
1.1. Some Linear Algebra and Differential Calculus	1
1.2. Differential Forms on an Open Subset $\Omega$ of $\mathbb{C}$	5
1.3. Partitions of Unity	15
1.4. Regular Boundaries	22
1.5. Integration of Differential Forms of Degree 2. The Stokes Formula	28
1.6. Homotopy. Fundamental Group	42
1.7. Integration of Closed 1-Forms Along Continuous Paths	56
1.8. Index of a Loop	66
1.9. Homology	71
1.10. Residues	84
1.11. Holomorphic Functions	91
CHAPTER 2	
Analytic Properties of Holomorphic Functions	98
2.1. Integral Representation Formulas	98
2.2. The Frechet Space $\mathcal{H}(\Omega)$	117
2.3. Holomorphic Maps	128
2.4. Isolated Singularities and Residues	138
2.5. Residues and the Computation of Definite Integrals	148
2.6. Other Applications of the Residue Theorem	165
2.7. The Area Theorem	178
2.8. Conformal Mappings	192
CHAPTER 3	
The $\bar{\partial}$ -Equation	213
3.1. Runge's Theorem	213

3.2. Mittag–Leffler’s Theorem	221
3.3. The Weierstrass Theorem	228
3.4. An Interpolation Theorem	235
3.5. Closed Ideals in $\mathcal{H}(\Omega)$	237
3.6. The Operator $\frac{\partial}{\partial \bar{z}}$ Acting on Distributions	245
3.7. Mergelyan’s Theorem	268
3.8. Short Survey of the Theory of Distributions. Their Relation to the Theory of Residues	278
CHAPTER 4	
Harmonic and Subharmonic Functions	299
4.1. Introduction	299
4.2. A Remark on the Theory of Integration	300
4.3. Harmonic Functions	304
4.4. Subharmonic Functions	318
4.5. Order and Type of Subharmonic Functions in $\mathbb{C}$	352
4.6. Integral Representations	366
4.7. Green Functions and Harmonic Measure	392
4.8. Smoothness up to the Boundary of Biholomorphic Mappings	413
4.9. Introduction to Potential Theory	433
CHAPTER 5	
Analytic Continuation and Singularities	480
5.1. Introduction	481
5.2. Elementary Study of Singularities and Dirichlet Series	482
5.3. A Brief Study of the Functions $\Gamma$ and $\zeta$	499
5.4. Covering Spaces	508
5.5. Riemann Surfaces	515
5.6. The Sheaf of Germs of Holomorphic Functions	523
5.7. Cocycles	534
5.8. Group Actions and Covering Spaces	541
5.9. Galois Coverings	544
5.10. The Exact Sequence of a Galois Covering	546
5.11. Universal Covering Space	550
5.12. Algebraic Functions, I	559
5.13. Algebraic Functions, II	565
5.14. The Periods of a Differential Form	572
5.15. Linear Differential Equations	583
5.16. The Index of Differential Operators	613
References	633
Notation and Selected Terminology	638
Index	646