

Contents

Chapter I. Riemann Surfaces	1
0 Basic Topological Notions	3
1 The Notion of a Riemann Surface	13
2 The Analytisches Gebilde	27
3 The Riemann Surface of an Algebraic Function	35
Appendix A. A Special Case of Covering Theory	46
Appendix B. A Theorem of Implicit functions	51
Chapter II. Harmonic Functions on Riemann Surfaces	54
1 The Poisson Integral Formula	56
2 Stability of Harmonic Functions on Taking Limits	60
3 The Boundary Value Problem for Disks	63
4 The Formulation of the Boundary Value Problem on Riemann Surfaces and the Uniqueness of the Solution	68
5 Solution of the Boundary Value Problem by Means of the Schwarz Alternating Method	74
6 The Normalized Solution of the External Space Problem Appendix. Countability of Riemann Surfaces	80 88
7 Construction of Harmonic Functions with Prescribed Singularities: The Bordered Case	91
8 Construction of Harmonic Functions with a Logarithmic Singularity: The Green's Function	95
9 Construction of Harmonic Functions with a Prescribed Singularity: The Case of a Positive Boundary	99
10 A Lemma of Nevanlinna	103
11 Construction of Harmonic Functions with a Prescribed Singularity: The Case of a Zero Boundary	113
12 The Most Important Cases of the Existence Theorems	118
13 Appendix to Chapter II. Stokes's Theorem	120
Chapter III. Uniformization	141
1 The Uniformization Theorem	143
2 A Rough Classification of Riemann Surfaces	152

VIII	Contents
3 Picard's Theorems	159
4 Appendix A. The Fundamental Group	163
5 Appendix B. The Universal Covering	170
6 Appendix C. The Monodromy Theorem	180
Chapter IV. Compact Riemann Surfaces	184
1 Meromorphic Differentials	184
2 Compact Riemann Surfaces and Algebraic Functions	192
3 The Triangulation of a Compact Riemann Surface	204
Appendix. The Riemann-Hurwitz Ramification Formula	208
4 Combinatorial Schemes	211
5 Gluing of Boundary Edges	218
6 The Normal Form of Compact Riemann Surfaces	222
7 Differentials of the First Kind	231
Appendix. The Polyhedron Theorem	237
8 Some Period Relations	238
Appendix. Piecewise smoothness	243
9 The Riemann-Roch Theorem	245
10 More Period Relations	254
11 Abel's Theorem	261
12 The Jacobi Inversion Problem	270
Appendix. Continuity of Roots	278
Appendices to Chapter IV	279
13 Multicanonical Forms	279
14 Dimensions of Vector Spaces of Modular Forms	285
15 Dimensions of Vector Spaces of Modular Forms with Multiplier Systems	295
Chapter V. Analytic Functions of Several Complex Variables	300
1 Elementary Properties of Analytic Functions of Several Variables	300
2 Power Series in Several Variables	302
3 Analytic Maps	311
4 The Weierstrass Preparation Theorem	317
5 Representation of Meromorphic Functions as Quotients of Analytic Functions	326
6 Alternating Differential Forms	337

Contents	IX
Chapter VI. Abelian Functions	347
1 Lattices and Tori	347
2 Hodge Theory of the Real Torus	352
3 Hodge Theory of a Complex Torus	355
4 Automorphy Summands	357
5 Quasi-Hermitian Forms on Lattices	364
6 Riemannian Forms	372
7 Canonical Lattice Bases	378
8 Theta Series (Construction of the Spaces $[Q, l, E]$)	384
Appendix. Complex Fourier Series	392
9 Graded Rings of Theta Series	393
10 A Nondegenerateness Theorem	396
11 The Field of Abelian Functions	403
12 Polarized Abelian Manifolds	408
13 The Limits of Classical Complex Analysis	412
Chapter VII. Modular Forms of Several Variables	427
1 Siegel's Modular Group	427
2 The Notion of a Modular Form of Degree n	431
3 Koecher's Principle	435
4 Specialization of Modular Forms	438
5 Generators for Some Modular Groups	441
6 Computation of Some Indices	449
7 Theta series	452
8 Group-Theoretic Considerations	459
9 Igusa's Congruence Subgroups	462
10 The Fundamental Domain of the Modular Group of Degree Two	466
11 The Zeros of the Theta Series of Degree two	469
12 A Ring of Modular Forms	475
Chapter VIII. Appendix: Algebraic Tools	483
1 Divisibility	483
2 Factorial Rings (UFD rings)	485
3 The Discriminant	488
4 Algebraic Function Fields	490
References	494
Index	499